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Exam I

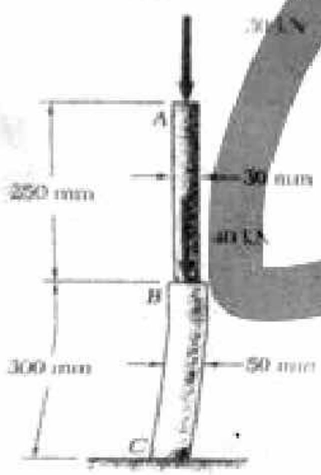
Wednesday March 27, 2013

All work **must** be shown to receive full credit

Problem 1. (20 pts.) Two solid cylindrical rods are joined and loaded as shown. Rod AB is made of steel ($E = 200\ GPa$) and rod BC of Brass ($E = 105\ GPa$). Determine:

a - The total deformation of the composite rod ABC.

b - The deflection of point B.



$$\delta_{AC} = \delta_{AB} + \delta_{BC}$$

$$= \frac{P_{AB} L_{AB}}{A_{AB} E_{AB}} + \frac{P_{BC} L_{BC}}{A_{BC} E_{AC}}$$

$$= \frac{-30\times10^3 \times 0.25}{\frac{\pi}{4}(0.03)^2 \times 200\times10^9} + \frac{-70\times10^3 \times 0.3}{\frac{\pi}{4}(0.05)^2 \times 105\times10^9}$$

$$= -1.55\times10^{-4}\ m$$

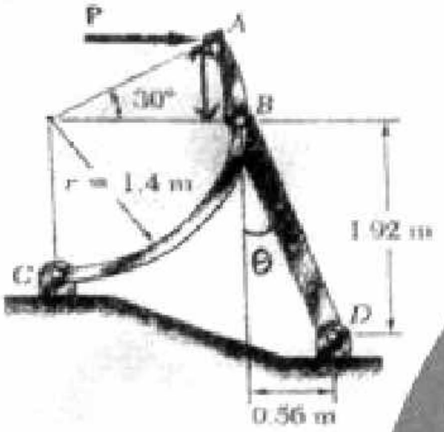
$$= -0.155\ m$$

$$\delta_B = \delta_{BC} = \frac{-70\times10^3 \times 0.3}{\frac{\pi}{4}(0.05)^2 \times 105\times10^9}$$

$$= -1.02\times10^{-4}\ m$$

$$= -0.102\ mm$$

Problem 2. (20 pts.) Knowing that the central portion of link BD has a uniform cross-sectional area of 800 mm^2, determine the magnitude of the load P for which the normal stress in that portion of BD is 50 MPa.



$$\sigma_{BD} = \frac{P_{BD}}{A_{BD}} \Rightarrow P_{BD} = \sigma_{BD} \times A_{BD}$$
$$= 50\times10^{6} \times 800 \times 10^{-6}$$
$$= 40kN = 40000\ N$$

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$$\tan\theta = \frac{0.56}{1.92}$$
$$\Rightarrow \theta = 16.26°$$

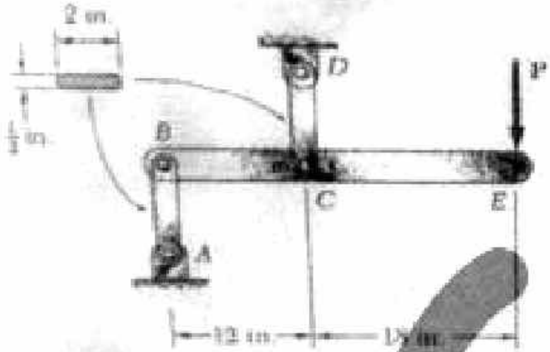
-2

-3

$$\sum M_C = 0 = F_{BD}\cos\theta\,(1.4) + F_{BD}\sin\theta\,(1.4) + P\,(1.4 + 1.4\tan 30)$$

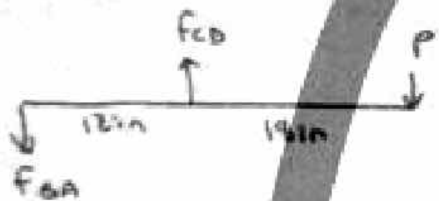
$$\Rightarrow 2.21\,P = -69439.86$$
$$\Rightarrow P = -31420.75\ N$$

Problem 3. (20 pts.) Each of the steel links AB and CD is connected to a support and to member BCE by 1 in diameter steel pins acting in single shear. Knowing that the ultimate shearing stress is 30 *ksi* for the steel used in the pins and that the ultimate normal stress is 70 *ksi* for the steel used in the links, determine the allowable load *P* if an overall factor of safety of 3.0 is desired.



$\tau_{all} = \frac{\tau_U}{n} = \frac{30}{3} = 10 ksi$

$\sigma_{all} = \frac{\sigma_U}{n} = \frac{70}{3} = 23.33 ksi$



$\Sigma M_B = 0 = 30P - 12F_{CD}$

$\Rightarrow F_{CD} = \frac{5P}{2}$

$\Sigma F_y = 0$

$F_{CD} - P - F_{BA} = 0$

$\Rightarrow F_{BA} = F_{CD} - P = \frac{5P}{2} - P = \frac{3}{2} P$

For normal stress:

Link CD: $\sigma_{all} = \frac{F_{CD}}{A} \Rightarrow 23.33 \times 10^3 = \frac{5P/2}{0.5 \times (2-1)}$

$\Rightarrow P = 4666 \text{ lb}$

Link BA: $\sigma_{all} = \frac{F_{BA}}{A} \Rightarrow 23.33 \times 10^3 = \frac{3/2\, P}{0.5(2-1)}$

$\Rightarrow P = 7776.67 \text{ lb}$

For shear stress:

Pin at C: $\tau_{all} = \frac{F_{CD}}{A} \Rightarrow 10 \times 10^3 = \frac{5/2\, P}{\pi/4\,(1)^2} \Rightarrow P = 3141.59 \text{ lb}$

Pin at B: $\tau_{all} = \frac{F_{BA}}{A} \Rightarrow 10 \times 10^3 = \frac{3/2\, P}{\pi/4\,(1)^2} \Rightarrow P = 5235.99 \text{ lb}$

$\Rightarrow \boxed{P = 3141.59 \text{ lb}}$

Problem 4. (20 pts.) Determine:

a - The compressive force in the bars shown after a temperature rise of 180° F

b - The corresponding change in length of the bronze bar.

Bronze	Aluminum
$A = 2.4$ in.^{2}	$A = 2.8$ in.^{2}
$E = 15 \times 10^6$ psi	$E = 10.6 \times 10^6$ psi
$\alpha = 12 \times 10^{-6}/°F$	$\alpha = 12.9 \times 10^{-6}/°F$

(a) $\delta_B + \delta_A = 0.02$

$$\frac{P L_B}{A_B E_B} + \alpha_B(\Delta T)L_B + \frac{P L_A}{A_A E_A} + \alpha_A(\Delta T)L_A = 0.02$$

~~P = 0.02 − (α_B ΔT L_B + α_A ΔT L_A)~~

$$\frac{P \times 14}{2.4 \times 15 \times 10^6} + 12\times10^{-6} \times 180 \times 14 + \frac{P \times 18}{2.8 \times 10.6 \times 10^6} + 12.9\times10^{-6}(180)(18) = 0.02$$

$\Rightarrow P = -52\,278.68$ N −2

(b) $\delta_B = \frac{P L_B}{A_B E_B} + \alpha_B(\Delta T)L_B$

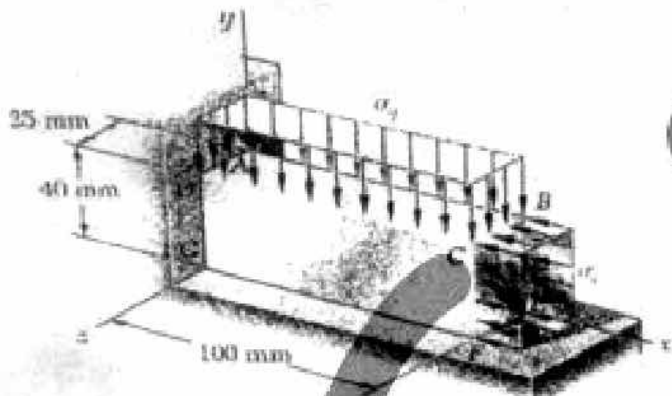
$$= \frac{-52278.68 \times 14}{2.4 \times 15 \times 10^6} + 12\times10^{-6} \times 180 \times 14$$

$= 9.9 \times 10^{-3}$ in.

Problem 5. (20 pts.) The block shown is made of a magnesium alloy for which $E = 45\ GPa$ and $\nu = 0.35$. Knowing that $\sigma_x = -180\ MPa$, determine:

a - The magnitude of σ_y for which the change in the height of the block will be zero.

b - The change in the area of the face $ABCD$.



(a) $\delta_y = y\left(\dfrac{\sigma_y}{E} - \dfrac{\nu\sigma_x}{E}\right)$

$0 = 0.04\left(\dfrac{\sigma_y}{45\times10^9} - \dfrac{0.35(-180\times10^6)}{45\times10^9}\right)$

$\Rightarrow \sigma_y = -63\ MPa$

(b) $\cancel{\delta_y = 0 \Rightarrow \delta_y}$

$\varepsilon_x = \dfrac{\sigma_x}{E} - \nu\dfrac{\sigma_y}{E}$

$= \dfrac{-180\times10^6 - 0.35\times(-63\times10^6)}{45\times10^9}$

$= -3.51\times10^{-3}$ but $\varepsilon_x = \dfrac{\delta x}{x} \Rightarrow \delta x = x\,\varepsilon_x$

$= 0.1(-3.51\times10^{-3})$

$= -3.51\times10^{-4}\ m$

$= -0.351\ mm$

$\delta z = z\left(\dfrac{-\nu\sigma_x}{E} - \dfrac{\nu\sigma_y}{E}\right)$

$= 0.025\left(\dfrac{0.35\times180\times10^6 + 0.35\times63\times10^6}{45\times10^9}\right) = 4.725\times10^{-5}\ m$

$= 0.04725\ mm$

~~$A_0 = x \times z = 2.5\times10^{-3}\ m^2$~~

~~$A' = (x+\delta x)(z+\delta z) = 2.49\times10^{-3}\ m^2$~~

~~$\delta A = \ldots\times10^{-3}\ m$~~

$\delta A = A' - A_0 = (x\cdot z) + (x+\delta x)(z+\delta z) - \ldots$